ISSN: 2581-8511
Tumbe Group of International Journal
A Peer Reviewed Multidisciplinary Journal
Volume- 4
Issue-2
May-August: 2021

A note on applications of Non-linear partial differential equations, and methods to solve

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Nowadays solving the Non-linear differential equations are challenging to everyone, meanwhile getting the analytical solution for those equations are too difficult. Many such governing differential equations are available in nature which governs the fluid flows. Governing equations of the fluid flow are coupled that is set of partial differential equations. These coupled equations are called equations of Navier-Stokes. These equations tell us how the pressure, velocity, temperature, and density of a moving fluid are related. In practical, solving these equations in analytically too difficult. The Navier-Stokes equations consists of conservation of mass, momentum and heat equations. Complexity of the non-linear system of partial differential equations which are not amenable to any of the known direct techniques. So we can prefer for the numerical methods such as FDM, FEM, FVM, Homotopy analysis Method(HAM), Perturbation Method, Adomain Decomposition Method(ADM), Shooting Method, Natural decomposition Method(NDM), and Runge Kutta Method etc. Computations are performed and numerical results are displayed graphically by using softwares such as Origion, Mathematica, MatLab, to illustrate the influence of the different models and different fluid parameters or different fluids with boundary layer of fluid flow. Then we can plot a simulated graph by varying physical flow parameters and non-dimensional numbers such velocity, temperature, concentration, Prandtl number, Reynolds number etc.

Keywords: Partial differential equations(PDE), Finite difference method (FDM), Newtonian and non-Newtonian fluid Prandtl number.

1. Introduction

Non-Newtonian fluids flows through various geometries have been innovative in recent decades, because of their functional application. Non-Newtonian fluids flows through various geometries have been innovative in recent decades, because of their functional application. In the nature many fluids are available such as Micro-polar fluids, Nanofluids, Couple stress fluids, Viscoelastic fluids etc. The numerous uses of non-Newtonian fluids in different fields, such as biological sciences, geophysics, petroleum, and chemical industries, have ignite such interest. Polluted air, as well as certain industrial fluids such as molten plastics, polymers, pulps, food, and fossil fuels, exhibit non-Newtonian behavior. Non Newtonian fluids have flow properties that are somewhat different from that of Newtonian fluids. Non-Newtonian fluids must be studied in order to gain a thorough understanding of these fluids and their applications. Many authors are studied different non-Newtonian fluid flow past a different geometries using different numerical methods [1-14]. Here author considered only Newtonian fluid with boundary condition for simplicity.

2. Mathematical formulation:

The time-dependent viscous incompressible boundary layer flow is considered. To describe the problem well, a rectangular cartesian coordinate system is implemented in which x-axis is measured along the axis of the plate and y-coordinate is taken perpendicular to the x-axis. More details about this physical flow situation with geometry can get in [15]. Under these assumptions, the boundary layer governing two-dimensional nonlinear equations of the present problem are described below with Boussinesq's approximation.

Continuity Equation: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

Momentum Equation: $$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta_T(T'-T'_\infty)++\nu\frac{\partial^2 u}{\partial y^2} \quad (2)$$

Energy Equation: $$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 u}{\partial y^2} \quad (3)$$

The corresponding initial and boundary conditions of the present investigated physical problem are as follows:

$$\begin{aligned}
&t'\le 0; \quad u=0,\ v=0,\ T'=T'_\infty \ \text{ for all } x \text{ and } y\\
&t'>0; \quad u=0,\ v=0,\ T'=T'_\infty \ \text{ at } y=0\\
&\qquad u=0,\ v=0,\ T'=T'_\infty \text{ at } x=0\\
&\qquad T'\to T'_\infty,\ u\to 0,\ v\to 0,\ \frac{\partial u}{\partial y}\to 0 \ \text{ as } y\to\infty
\end{aligned} \quad (4)$$

Now introduce the following non-dimensional variables and parameters are,

$$\begin{aligned}
&X=Gr_T^{-1}\frac{x}{l}\quad Y=\frac{y}{l}\ U=Gr_T^{-1}\frac{ul}{\nu}\quad V=\frac{vl}{\nu}\quad t=\frac{\nu t'}{l^2}\\
&T=\frac{T'-T'_\infty}{T'_w-T'_\infty}\qquad Pr=\frac{\nu}{\alpha}\qquad Gr_T=\frac{g\beta_T l^3(T'_w-T'_\infty)}{\nu^2}
\end{aligned} \quad (5)$$

By introducing the above non- dimensional variables and parameters in the equations (1), (2) and (3) then the following dimensionless equations are obtained.

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (6)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+T \quad (7)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2} \quad (8)$$

The corresponding Initial and Boundary conditions in non-dimensional variables are given as follows

$$\begin{aligned}
&t\le 0; \quad T=0, C=0, U=0,\ V=0,\ \text{ for all } X \text{ and } Y\\
&t>0; \quad T=1, C=1, U=0, V=0 \ \text{ at } Y=0\\
&\qquad T=0,\ C=0, U=0,\ V=0 \ \text{ at } X=0\\
&\qquad T\to 0,\ \ C\to 0,\ \ U\to 0,\ V\to 0,\ \frac{\partial U}{\partial Y}\to 0 \ \text{ as } Y\to\infty
\end{aligned} \quad (9)$$

3. Numerical Approach

To solve above non-dimensional system of partial differential equations along with boundary conditions author used the finite difference method (FDM). After implementing of this method, the system is converted as Tridiagonal form. Initially the temperature field is obtained by solving the energy Eq. (8). The velocity U is computed from Eq. (7) by using known values of T. The velocity V is calculated from the finite difference equation of Eq. (6) explicitly. The more details application of this method can be found in Ref. [16] and cited therein.

4. Result and discussion

Effect of Prandtl number

Velocity field: The simulated velocity field U verses t as shown in the Fig. 1(a). It is observed that velocity field declining by increasing the Prandtl number, Pr. Initially it seems that all U profiles are merging with each other due to conduction dominates the convection of heat processes. Further, as time progress the velocity profile attains its maxima, decreases and finally turns to steady-state asymptotically. The Fig. 1(b) indicates the steady-state velocity profile with varying values of Pr. As noticed from the graph that, initially velocity flow field started with zero, reaches its maximum values and decreases monotonically to zero along with Y direction. Further, the observation here is that, as Pr increases, the steady state velocity profiles are decreases.

Temperature field: The unsteady temperature field as shown in Fig. 2(a). As noticed from the figure, temperature filed decreases as enhancing Pr. Further, temperature field attains its maxima , shortly later diminishes and reach the steady state. The Fig. 2(b) describe the steady state temperature profile with different values of Pr. The temperature curves are starts with hot vertical wall at $T = 1$ and decreases monotonically to zero for the time. However, steady state temperature profile decreases as raising Pr.

5. Conclusion

The proposed research article examines a note on applications of partial differential differential equations. In this regard simulated results of system of PDE's which governs the Newtonian fluid solved by applying FDM procedure by varying Prandtl number Pr. From the present analysis we have made following conclusions

- Unsteady and steady Velocity profile diminishes by enhancing Pr.
- Unsteady and steady Temperature profile diminishes by enhancing Pr.

6. Future work

The main objectives in this research are.

- To Investigation of Newtonian and non-Newtonian(nano) fluids with boundary layer flows by numerical and analytical methods with comparison between these two methods.
- To study the problems by considering different geometries like Stretching sheet, Horizontal or Vertical plates, Cylinders, open channels etc and models.
- To study the problems with different physical effects like MHD, Thermal radiation, Energy dissipation, Chemical reaction, porous medium, etc.
- To investigate various mathematical models in order to obtain the effect of various physical parameters on fluid flow.

List of symbols

u, v: velocity components in $(x,\ y)$ coordinate system
t' : time
t: dimensionless time
T' : temperature
T : dimensionless temperature
C_p : specific heat at constant pressure
g: acceleration due to gravity
Gr_T: thermal Grashof number
Pr: Prandtl number
k: thermal conductivity
y: normal coordinate measured from the axis of the plate
x: axial coordinate measured vertically upward
U, V: dimensionless velocity components in $(X,\ Y)$ coordinate system
X: dimensionless axial coordinate
Y: dimensionless normal to X coordinate
α : thermal diffusive coefficient

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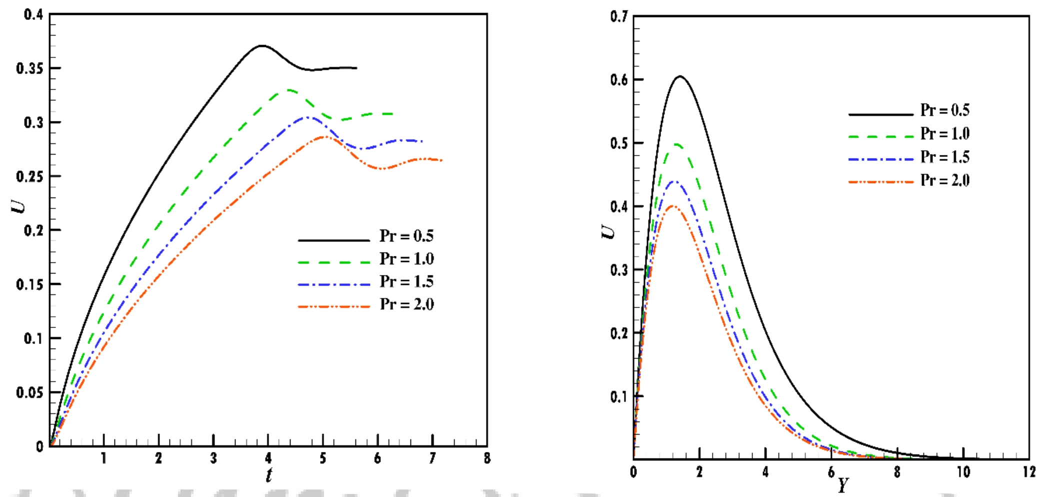
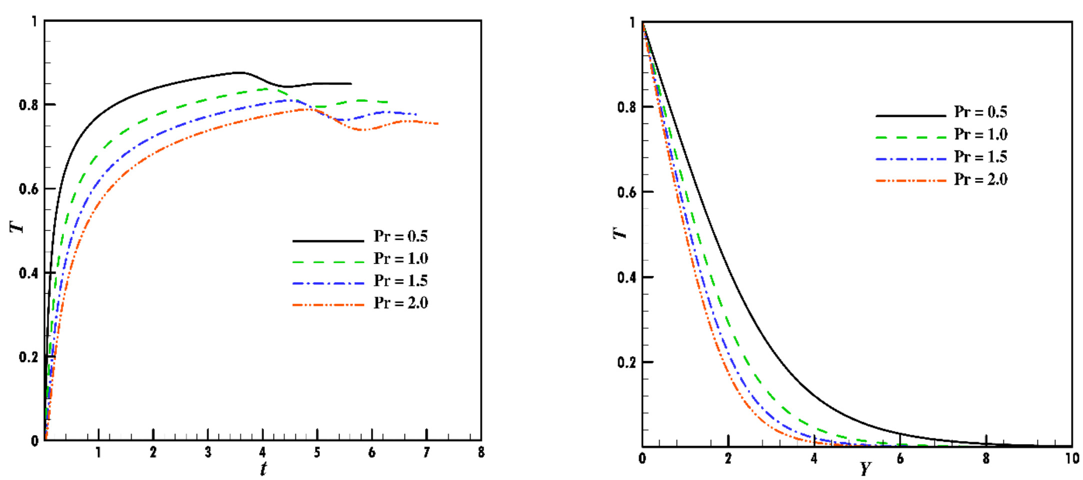


(a) (b)

Fig. 1: Velocity profiles for various values of Pr (a) Transient (b) Steady-state.



(a) (b)

Fig. 2: Temperature profiles for various values of Pr (a) Transient (b) Steady-state.