



## NEAR FELICITOUS DIFFERENCE LABELING GRAPH

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### Abstract

A simple graph  $G$  is called Near Felicitous if there exists a 1 – 1 function  $f: V(G) \rightarrow \{0,1,2, \dots, q-1, q+1\}$  in such that the set of induced edges labels  $f^*(xy) = (f(x) + f(y)) \pmod{q+1}$  are all distinct when the subtraction is taken modulo  $q+1$  with residues  $\{1,2, \dots, q\}$ . In this paper, we prove that Path, Complete Bipartite Graph, Fan, Ladders ( $n \geq 2$ ), Peterson are Near Felicitous Difference Labeling Graph. As a consequence, some families of graphs are shown to be non – near felicitous difference labeling.

**Keywords:** Near Felicitous Difference Labeling (NFDL), Path, Cycle, Complete Bipartite Graph, Fan, Ladders, Peterson Graph

### I. INTRODUCTION

In 1966, Rosa [3] introduced  $\alpha$  - valuation of a graph and subsequently Golomb introduced graceful labeling. In 1980, Graham and Slonae [2] introduced the harmonious labeling of a graph. Several graph labelings have been found in Gallian survey [1]. Lee and schmeichel and shee [4] introduced the concept of felicitous graph as a generalization of harmonious graph. V. Lakshmi Alias Gomathi, A. Nagarajan, A. Nellai Murugan introduced the concept of felicitous labeling of a graph [5]. A Felicitous Difference Labeling of a graph  $G$ , with  $q$  edges is an injection  $f : V(G) \rightarrow \{0,1,2, \dots, q\}$  in such a way that each edge  $e = uv$ , is labeled as  $f^*(xy) = (f(x) + f(y)) \pmod{q+1}$ . The resulting labels of the edges are distinct and form  $\{1,2, \dots, q\}$ .

Near Felicitous labeling graph was introduced by V. Lakshmi Alias Gomathi, A. Nagarajan, A. Nellai Murugan[6]. It motivates us to define the concept of near felicitous difference labeling graph.

### II. Preliminaries

**Definition 2.1:** A Path  $P_n$  is a walk in which all the vertices are distinct.

**Definition 2.2:** A Cycle  $C_n$  is a Closed Path.

**Definition 2.3:** A Complete Bipartite graph  $k_m$  is a bipartite graph with bipartition  $(V_1, V_2)$  such that every vertex of  $V_1$  is joined to all the vertices of  $V_2$ , Where  $|V_1| = m$  and  $|V_2| = n$ .

**Definition 2.4:** The fan  $f_n$  ( $n \geq 2$ ) is obtained by joining all vertices of  $P_n$  (Path of  $n$  vertices) to a further vertex called the center and contains  $n + 1$  vertex and  $2n - 1$  edges. i.e.  $f_n = p_n + k_1$ .

**Definition 2.5:** The ladder  $L_n$  ( $n \geq 2$ ) is the product graph  $p_2 \times p_n$  which contains  $2n$  vertices and  $3n - 2$  edges.

**Definition 2.6:** Peterson graph with 10 vertices and 15 edges. It is most commonly drawn as pentagon with a pentagram inside with five spokes.

### III. Main Results

**Definition 3.1:** A Felicitous Difference Labeling of a graph  $G$ , with  $q$  edges is an injection  $f : V(G) \rightarrow \{0, 1, 2, \dots, q - 1, q + 1\}$  in such a way that each edge  $e = uv$ , is labeled as  $f^*(xy) = (f(x) + f(y)) \pmod{q + 1}$ . The resulting labels of the edges are distinct and form  $\{1, 2, \dots, q\}$ . A graph that admits a **Near Felicitous Difference Labeling (FDL)** is called **Near Felicitous Difference Labeling Graph**.

**Example: 3.1.1** NFD labeling of the cycle  $c_8$  is given in fig. 1

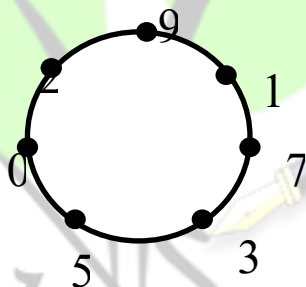


Fig. 1

**Theorem 3.2:** Any Path  $P_n$  is a NFDL.

**Proof:** Let  $V(P_n) = \{u_i / 1 \leq i \leq n\}$  and  $E(P_n) = \{u_i u_{i+1} / 1 \leq i \leq n-1\}$ . Then  $|V(P_n)| = n$  and  $|E(P_n)| = n - 1$ . Define  $f: V(P_n) \rightarrow \{0, 1, 2, \dots, n-2, n\}$  as follows:

$$f(u_1) = n, \quad f(u_2) = 1, \quad f(u_3) = n - 2, \quad f(u_4) = 0, \quad f(u_5) = n - 4,$$

$$f(u_6) = \begin{cases} 2, & n \geq 7 \\ 3, & n = 6, 8 \end{cases}, \quad f(u_7) = n - 3,$$

$$f(u_8) = \begin{cases} 2, & n = 8 \\ 4, & n \geq 9 \end{cases}, \quad f(u_9) = \begin{cases} 3 & n = 9 \\ 5 & n = 10, 11 \\ 7 + h & n \geq 12, h = 0, 1, 2, 3, \dots \end{cases}$$

$$f(u_{10}) = 3$$

$$f(u_{11}) = \begin{cases} 5 & n = 12 \\ 6 & n = 11, 12 \\ 7 + h & n \geq 14 \end{cases}, \quad f(u_{12}) = \begin{cases} 5, & n \geq 13 \\ 6, & n = 12, 14 \end{cases}$$

$$f(u_{13}) = 7 + h, \quad f(u_{14}) = f(x) = \begin{cases} 5, & n = 14 \\ 7, & n \geq 15 \end{cases}$$

Continuing in this way, we get the induced edge labels  $f^*E(G) = \{1, 2, \dots, n\}$  are all distinct.

**Example 3.3:** The FD labeling of  $P_6$  and  $P_{15}$  are given in fig. 1 and 2 respectively.

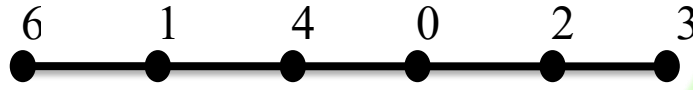


Fig. 2

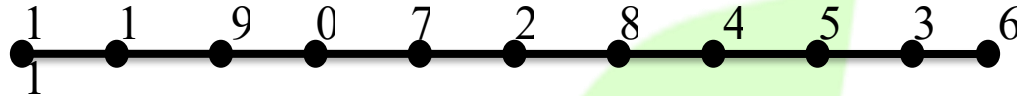


Fig. 3

**Theorem 3.4:** For each  $m, n \geq 1$ , the complete bipartite graph  $K_{m,n}$  is NFDL graph.

**Proof:** Let the bipartition of  $K_{m,n}$  be  $(X, Y)$  where  $X = \{x_1, x_2, \dots, x_m\}$  and  $Y = \{y_1, y_2, \dots, y_n\}$ .

Assume that,  $m \leq n$ . But  $m \neq n \neq 1$ . Then, a near felicitous labeling of  $K_{m,n}$  is

$$f(y_0) = mn + 1$$

$$f(x_i) = i, \quad i = 1, 2, \dots, m \text{ and}$$

$$f(y_i) = y_{i-1} - m, \quad 1 \leq i \leq n$$

Then the induced edge labels are distinct. Hence  $K_{m,n}$  admits NFDL graph.

**Example 3.5:** The NFD labeling of  $K_{2,4}$  are given in fig 5 respectively.

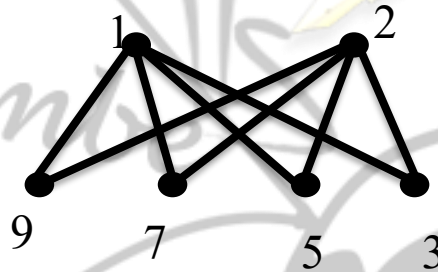


Fig. 4

**Theorem 3.6:** For a fan graph,  $f_n$  is NFDL graph for  $n \geq 2$ .

**Proof:** Let  $u_0$  be the centre of fan. Define  $f: V(f_n) \rightarrow \{0, 1, 2, \dots, 2n - 3, 2n\}$  as follows:

$$f(u_0) = 2n$$

$$f(u_{2i+1}) = i, \quad 1 \leq i \leq n$$

$$f(u_{2i}) = n - i, \quad 0 \leq i \leq n$$

let  $f^*$  be the induced edge labeling of  $f$ . Then

$$f^*(u_0, u_i) = n + i, \quad 0 \leq i \leq n$$

$$f^*(u_i, u_{i+1}) = n - i, \quad 1 \leq i \leq n$$

Then the induced edge labels are all distinct. Hence from the above labeling pattern, the graph admits  $f_n$  NFDL graph.

**Example 3.7:** The NFD labeling of  $f_3$  are given in fig 6 respectively.

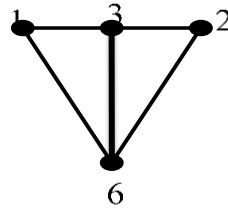


Fig. 5

**Theorem 3.8:** All ladders  $n \geq 2$  are NFDL graph.

**Proof:** Define  $f: V(l_n) \rightarrow \{0,1,2, \dots, 3n-3, 3n-1\}$  is defined as follows:

$$f(u_1) = 1$$

$$f(u_{2i+1}) = u_{2j+1} + 3, \quad 1 \leq i \leq n, 0 \leq j \leq n$$

$$f(u_2) = 2$$

$$f(u_{2i}) = u_{2j} + 3, \quad 2 \leq i \leq n, 1 \leq j \leq n$$

$$f(v_1) = 3n - 1$$

$$f(v_{2i+1}) = v_{2j+1} - 3, \quad 1 \leq i \leq n, 0 \leq j \leq n$$

$$f(v_2) = 3n - 3$$

$$f(v_{2i}) = v_{2j} + 3, \quad 2 \leq i \leq n, 1 \leq j \leq n$$

Then the induced edges labels are distinct. Hence  $l_n$  admits is a NFDL graph.

**Example 3.9:** The NFD labeling of  $l_4$  are given in fig 7 respectively.

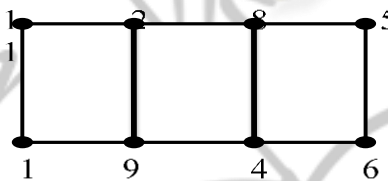


Fig. 6

**Theorem 3.10:** The Peterson graph is NFDL graph. The following fig. 7

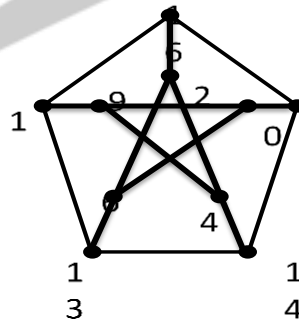


Fig. 7

**Results 3.11:**  $C_n$  is a NFDL graph if  $n = 4k$ ,  $k = 1, 2, 3, \dots$

**Results 3.12:** Every Star graph is FDL graph. But Every Star graph is not NDFL Graph.

**Results 3.13:** Let  $G$  be a graph with odd number of edges and  $f: V(G) \rightarrow \{0, 1, 2, \dots, q-1, q+1\}$  be a near odd edge labeling of  $G$ . Then  $f$  is not near felicitous labeling of  $G$ .

**Results 3.14:** Let  $G$  be a graph with even number of edges and  $f: V(G) \rightarrow \{0, 1, 2, \dots, q-1, q+1\}$  be a near even edge labeling of  $G$ . Then  $f$  is not near felicitous labeling of  $G$ .

**Results 3.15:** Let  $G$  be a graph with even number of edges and  $f: V(G) \rightarrow \{0, 1, 2, \dots, q-1, q+1\}$  be a near odd edge labeling of  $G$ . Then  $f$  is not near felicitous labeling of  $G$ .

### III. Conclusion

The Study of labeled graph is important due to its diversified applications. All graphs are not NFDL graphs. It is very interesting to investigate graphs which admit NFD Labeling. In this paper, we proved that Path, Complete Bipartite Graph, Fan, Ladders ( $n \geq 2$ ), Peterson graph are Near Felicitous Difference Labeling Graph. We have already investigated graphs which are NFDL graph only for certain cases and have planned to investigate the NFD labeling of some special cases of path and cycle related graphs in our next paper.

### IV. References

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