An Alpha-Series Process Repair Model for Repairable System with Different Repair Actions

[1]Dr. K. RAJA SEKHARA REDDY and [2]Dr. V SIVA PRASAD

[1]assistant Professor, Government First Grade College, Pavagada – 561202.
[2]assistant Professor, Government First Grade College, Channapatna – 562160.

ABSTRACT

In this paper, we investigate the maintenance problem for a cold standby system with two dissimilar components and one repairman with two repair actions is studied. Assume that the component 1 follow α-series process repair and is given priority in use while component 2 obeys a minimal repair. Under these assumptions, using an alpha series process repair model, we consider a replacement policy N under which the system is replaced when the number of failures of component 1 reaches N. The purpose of this chapter is to determine an optimal replacement policy N* such that the average cost rate (i.e. the long-run average cost per unit time) of the system is minimized. The explicit expression for the average cost rate of the system is derived and the corresponding optimal replacement policy N* can be determined analytically or numerically. Finally, a numerical example is given to illustrate some theoretical results and the model applicability.

Keywords: Alpha Series Process Renewal Reward Theorem Replacement Policy Renewal Process Long Run Average Cost Rate.

1. INTRODUCTION

In maintenance problems, the standby technique is often adopted to improve the system reliability, availability and reduce the cost incurred due to system failure. In the beginning, Lotka introduced a repair–replacement model which is extensively used to study a one-component repairable system with one repairman. Moreover, it is usually assumed that the system after repair is ‘‘as good as new”. The model under this assumption is called as a perfect repair model. However, this assumption is not always true. In practice, most repairable systems are deteriorative because of the ageing effect and the accumulative wear. Barlow and Hunter presented a minimal repair model in which a system after repair has the same failure rate and the same effective age as at the time of failure. Brown and Proschan, combined the perfect repair and the minimal repair, which is proposed as an imperfect repair model. In this model they assumed that the repair will be perfect with probability ‘p’ or minimal with probability ‘1-p’. Much Research works on the minimal repair model and the imperfect repair models were studied. However, for a deteriorating simple system, it is more reasonable to assume that the successive working times of the system after repair will become shorter and shorter, while the consecutive repair times of the system after failure will become longer and longer. Ultimately, it neither works any longer nor repairs any more.

For such a stochastic phenomenon, Lam introduced a geometric process repair model by assuming that the system after repair is not “as good as new” and the successive working times are stochastically decreasing while, the successive repair times are stochastically increasing in the long run..Under this model, he studied two kinds of replacement policy for simple repairable systems, one based on the working age T of the systems and the other based on the failure number N of the system. The explicit expressions of the average cost rate under these two kinds of policy are respectively calculated, and the corresponding optimal replacement policies T* and N* can be find analytically or numerically. Under some mild conditions, he also proved that the optimal policy N* is better than the optimal policy T*.

Yuan Lin Zhang and Guan Jun Wang analyzed a two-component cold standby repairable system with one repairman. It assumed that each component after repair was not ‘‘as good as new”, and obeys a geometric process repair. Under this assumption, he studied a replacement policy N based on the number of repairs of component 1. An optimal replacement policy N* can be determined by maximizing the long-run expected reward per unit time.

Yuan Lin Zhang investigated a repairable system consisting of one component and a single repairman (i.e. a simple repairable system) with delayed repair. It is assumed that the working time distribution, the repair time distribution and the delayed repair time distribution of the system are all exponential. After repair, the system is not “as good as new”. Under these assumptions, by using the geometrical process and the supplementary variable technique, they derived some important reliability indices such as the system availability, rate of occurrence of failures (ROCOF), reliability and mean time to first failure (MTTFF). A repair replacement policy N under which the system is replaced when the number of failures of the system reaches N is also studied. The explicit expression for the average cost rate (i.e. the long-run average cost per unit time) of the system is derived, and the corresponding optimal replacement policy N ∗ can be found analytically or numerically. Finally, a numerical example for policy N is given.

Similarly, in order to improve the reliability, raise the availability or reduce the cost of a system, the techniques for priority in use or repair were also used. For example, in the operating room of a hospital, an operation must be discontinued if only the power source is cut (i.e. the power station fails).Usually, there is a standby generator (e.g. a storage battery) which can provide electric power when the main power station fails. Thus, the power station (regarded as the main component, written as component 1) and the storage battery (as the cold standby component, written as component 2) form a repairable electricity-supply system. Obviously, it is reasonable to assume that the power station has use priority due to the operating cost of the power station is cheaper than the storage battery and the electricity-supply systems in a hospital are the some similar examples.

Nakagawa assumed that both working time and repair time of the priority component follow general distributions while both working time and repair time of the non- priority component follow exponential distributions, and the repairs are perfect. Under these assumptions, By using the Markov renewal theory, they developed some interesting reliability indices for the system. Brown et.al studied some important properties of monotone processes and proved that alpha series processes is more appropriate to model the up times and proved that the second ordered moment does not exist for uptimes.

Further, Guan Jun Wang and Yuan Lin Zhang studied a two component dissimilar-component cold standby system with different repair actions under the assumptions that the component 1 has priority in use and the successive working times component 1 form a decreasing geometric process while, the consecutive repair time forms a general repair process and component 2 obeys a minimal repair. Under these assumptions, using geometric process repair model, they consider a replacement policy N under which the system is replaced when the number of failures of component 1 reaches N. An optimal replacement policy N* is determined such that the average cost rate (i.e. the long-run average cost per unit time) of the system is minimized. The explicit expression for the average cost rate of the system is derived and the corresponding optimal replacement policy N* can be determined analytically or numerically. Finally, a numerical example is given to illustrate some theoretical results and the model applicability.

In this chapter, the α-series process repair model for a two-dissimilar component cold stand by repairable system with one repairman is studied by assuming that each component after repair is not ‘‘as good as new” and follows a α-series process repair, and component 1 has use priority and component2 obeys minimal repair. A repair–replacement policy N based on the number of failures of component 1 under which the system is replaced when the failure number of component 1 reaches N is studied .The aim is to determine an optimal replacement policy N* such that the average cost rate of the system is minimized and derived an explicit expression for the average cost rate of the system. Finally, a numerical example is given to illustrate some theoretical results included the uniqueness of the optimal replacement policy N*.

2. The Model

To study the maintenance problem for a two-dissimilar-component cold stand by repairable system with different repair actions, the following assumptions are imposed.

Assumption1: At the beginning, the two components are both new and component1 is in a working state while component 2 is in a cold standby state.

Assumption2: Assume that the component 1 after repair is not ‘‘as good as new” and follow a α-series process repair. When both components are good, component1 has use priority.

Assumption3: The time interval between the completion of the (n-1)th repair and the completion of the nth repair of the component 1 called nth cycle (i.e., the nth repair cycle) of component. Note that a component either begins to work or enters a cold stand by state of the next cycle when its repair is completed. Because component1 has use priority, there pair time of component2 may be zero in some cycles.

Assumption4: When component 2 fails, a minimal repair is performed that returns the component to the function state just before failure.

Assumption5: Assume that the working time of the component 2 having the distribution function H(Z) with failure r(t).The repair time of component 2 is negligible.

Assumption6: Let X_n and Y_n be respectively, the working time and the repair time of component ‘1’ in the nth cycle. Then the distribution functions of X_n and Y_n are respectively given by $F_n(x)$ and $G_n(y)$ .Where

$$F_n(x) = F(k^{\alpha} x), \quad G_n(y) = G(k^{\beta} y) \quad where \ x, y \geq 0, \ \ a_i > 0, \ \beta < 0 \ .$$

Assumption7: A sequence { X_n ,n=1,2,....}and a sequence { Y_n ,n=1,2,....}and the lifeZ are mutually independent .

Assumption8: There placement policy N based on the number of failures of component1 is used. The system is replaced by a new and identical one at the N^{th} failure of component1,and there placement time is negligible.

Assumption9: There pair cost rate of component is $C_i, i=1,2$ while the working reward rate of two components is same C_w. And there placement cost of the system is C.

3. Average Cost Rate under Policy N

According to the assumptions of the model ,the two components appear alternately in the system. When the failure number of component1 reaches N, component2 is in the cold standby state of the N^{th} cycle as the repair time of component 2 is negligible. Naturally, a reasonable replacement policy N should be that if component2 is in the working state, it was not works until failure in the N^{th} cycle as component 1 has use priority. Thus, the renewal point under the policy N is efficiently established.

Let s_1 be the first replacement time of the system, and s_n (n>2) be the time between the (n-1)th replacement and the nth replacement of the system under policy N. Thus, $\{s_1,s_2,\ldots\}$ forms a renewal process, and the inter arrival time between two consecutive replacements is called a renewal cycle.

Let C(N) be the average cost rate of the system under policy N. According to renewal reward theorem, then

$$C(N)=\frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length in a renewal cycle}} \qquad (2.1)$$

Because component 1 has use priority, component 1 only exist the working state. Therefore based on the renewal point under policy N we have the following length of renewal cycle. Let W(N) and R(N) be the total working time and total repair time of the system during a renewal cycle. That is

$$L=W(N)+R(N)$$
$$L=\sum_{k=1}^{N}X_k^{(i)}+\sum_{k=1}^{N-1}Y_k^{(i)} \qquad (2.2)$$

Since $Y_1, Y_2, \ldots\ldots Y_{N-1}$ are independent and identically distributed,

Let $R(N)=Y_1+Y_2+\ldots\ldots+Y_{N-1}$ is follows $G^{(N-1)}(t)$.

Where $G^{(N-1)}(t)$ is the (N-1) fold convolution of the distribution G.

Denote N(t), the failure number of component 2, in the time interval (0,t] when it is operating in (0,t] with the minimal repair. Then {N (t), t>=0} will constitute a non-homogenous Poisson process with failure intensity r(s).Thus, the mean of N(t) is

$$E[N(R(N)]=\int_0^t r(s)\,ds \qquad (2.3)$$

Because the total working time of component 2 in a renewal cycle is $R(N) = Y_1 + Y_2 + ... + Y_{N-1}$, the failure number of component 2 in a renewal cycle is N(R(N)), and its expected value is

$$E[N(R(N)] = \int_0^\infty E[N(t)]d\, G^{(n-1)}(t) \tag{2.4}$$

$$= \int_0^\infty \left[\int_0^t r(s)ds\right] d\, G^{(n-1)}(t) \tag{2.5}$$

$$= \int_0^\infty r(s)\ \overline{G}^{(n-1)}(s)ds \tag{2.6}$$

Where $\overline{G}^{(N-1)}(s) = 1 - G^{(N-1)}(s)$ is the survival function of $R(N) = Y_1 + Y_2 + ... + Y_{N-1}$.

Let C(N) be the long-run average cost rate (ACR) of the system under policy N. According to the renewal reward theorem [see Ross(1970)],

$$C(N) = \frac{C + C_1 E\left(\sum_{k=1}^{N-1} Y_k\right) + C_2 E[N(R(N)]}{E\left(\sum_{k=1}^{N} X_k\right) + E\left(\sum_{k=1}^{N-1} Y_k\right)} \tag{2.7}$$

Now, we evaluate the expected value of Xn and Yn and N(R(N) as follows:

According to the assumption of the model, we have

If $\{X_n, n=1,2,....\}$ is distributed according to an exponential failure law with distribution function $F_n(n^\alpha x)$, then we have

$$E(X_n) = \int_0^\infty x d F(n^\alpha x) \tag{2.8}$$

$$= \frac{\lambda}{k^\alpha},\ \alpha > 1, \lambda > 0. \tag{2.9}$$

If $\{Y_n, n=1,2,....\}$ is distributed according to an exponential failure law with distribution function $F_n(n^\beta x)$, then we have

$$E(Y_n) = \int_0^\infty y d F(k^\beta y)\,. \tag{2.10}$$

$$= \frac{\mu}{k^\beta},\ \beta < 0, \mu > 0. \tag{2.11}$$

According to assumption (4) , the distribution function of Z is given by

$$H(z) = 1 - \exp(-z/\theta)\quad . \tag{2.12}$$

Then failure intensity is given by

$$r = \frac{1}{\theta}\quad . \tag{2.13}$$

If the life time of component 2 is exponentially distributed with constant rate r, then the failure number $\{N(t), t \geq 0\}$ of component 2 form a homogeneous Poisson process with intensity r. Thus the expected failure number of component 2 in a renewal cycle is given by

From equation (3.6) we have

$$E[N(R(N)] = \int_0^\infty r\, \overline{G}^{(n-1)}(s)ds \;. \qquad (2.14)$$

Since, $Y_1, Y_2, \ldots\ldots Y_{N-1}$ are independent and identically exponentially distributed then the survival function $\overline{G}^{(N-1)}(s) = 1 - G^{(N-1)}(s)$ is given by

$$P(Y_1 + Y_2 + \ldots\ldots + Y_{N-1} > t) = \sum_{k=0}^{n-1} \left(\frac{\left(\frac{k^\beta t}{\mu}\right)^k e^{-\left(\frac{k^\beta t}{\mu}\right)}}{k!} \right). \qquad (2.15)$$

From equation (3.14) and (3.15), we have

$$E[N(R(N)] = \int_0^\infty r \sum_{k=1}^{n-1} \left(\frac{\left(\frac{k^\beta t}{\mu}\right)^k e^{-\left(\frac{k^\beta t}{\mu}\right)}}{k!} \right) dt. \qquad (2..16)$$

From equation (3.13), we have

$$E[N(R(N)] = \int_0^\infty \frac{1}{\theta} \sum_{k=1}^{n-1} \left(\frac{\left(\frac{k^\beta t}{\mu}\right)^k e^{-\left(\frac{k^\beta t}{\mu}\right)}}{k!} \right) dt.$$

$$E[N(R(N)] = \frac{\mu}{\theta} \sum_{k=1}^{n-1} \frac{1}{k^\beta} . \qquad (2.17)$$

From equations (2.9),(2.11) and (2.17) ,the equation (2.1) becomes

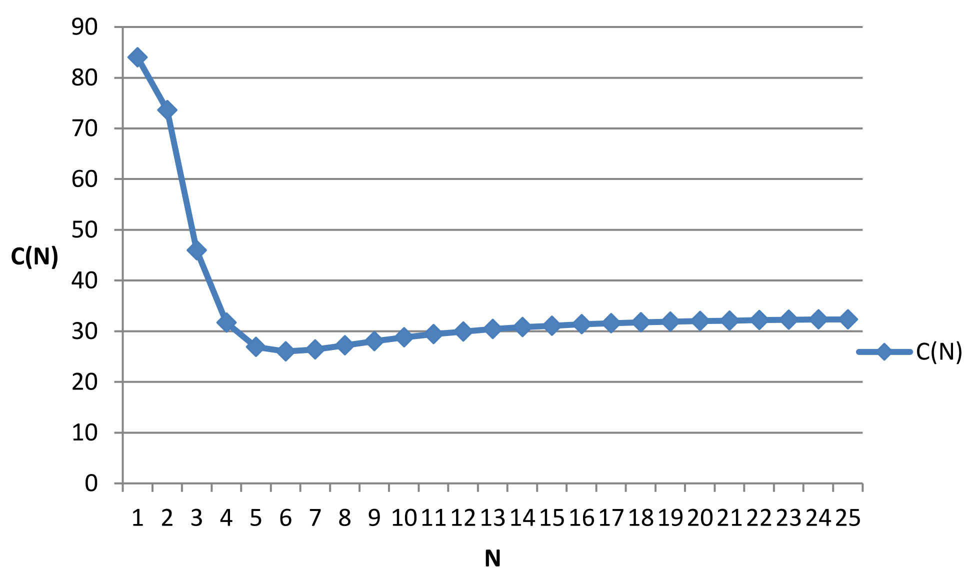
$$C(N) = \frac{C + C_1 \sum_{k=1}^{N-1} \frac{\mu}{k^\beta} + C_2 \frac{\mu}{\theta} \sum_{k=1}^{N-1} \frac{1}{k^\beta}}{\sum_{k=1}^{N} \frac{\lambda}{k^\alpha} + \sum_{k=1}^{N-1} \frac{\mu}{k^\beta}} . \qquad (2.18)$$

4. Numerical Results and Conclusions

For the hypothetical values C_r=40, C=2500 , Cw=100, p=0.75, α=0.95, β=-0.95, λ =20, μ=30 it could be computed the long-run average cost per unit of time is computed as follows:

Table- 4.1: **The long-run average cost per unit of time values against N**

N	C(N)	N	C(N)
1	84	14	30.78261983
2	73.58611416	15	31.09462423
3	45.96808144	16	31.35253205
4	31.71588738	17	31.56544718
5	26.93002094	18	31.7410108
6	**26.01872761**	19	31.88559439
7	26.42633907	20	32.00449838
8	27.20515676	21	32.10213054
9	28.02162693	22	32.18215759
10	28.76572121	23	32.24763108
11	29.40895758	24	32.30109101
12	29.95217047	25	32.34465099
13	30.40583785		

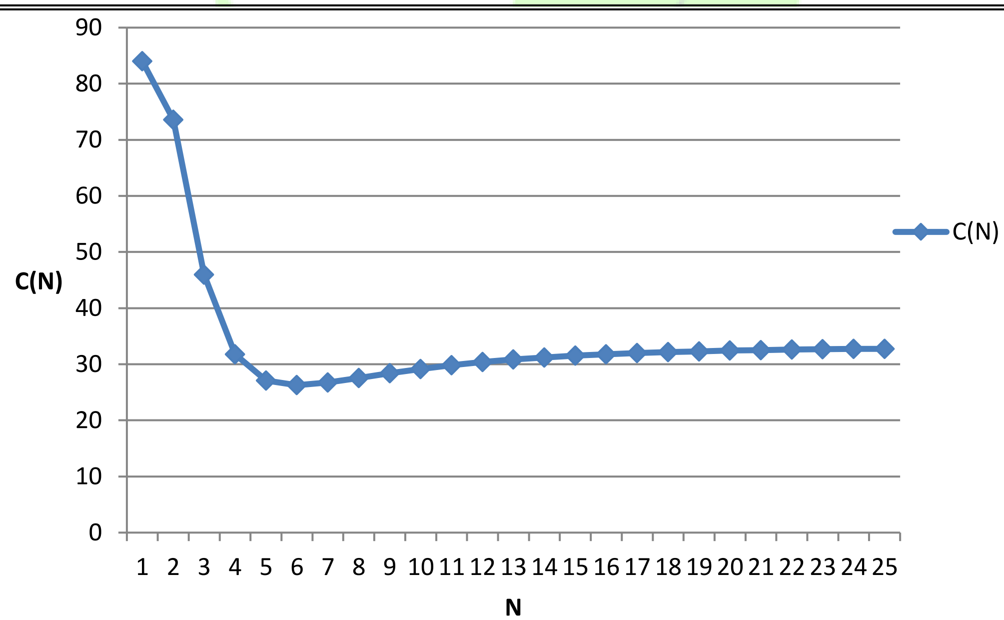


Graph 4.1: The long-run average cost per unit of time values against N

Table 4.2: **The long-run average cost per unit of time values against N**

For the hypothetical values C_r=40, C=2500,C_w=100, p=0.7, α=0.95, β=-0.99,λ =20 ,μ=30 it could be computed the long-run average cost per unit of time is computed as follows:

N	C(N)	N	C(N)
1	84	14	31.21485235
2	73.58611416	15	31.52688167
3	45.96318943	16	31.78443134
4	31.75998622	17	31.99683456
5	27.08444256	18	32.17186243
6	**26.27475889**	19	32.31595504
7	26.75367527	20	32.43444417
8	27.57765535	21	32.53174693
9	28.42137252	22	32.61152703
10	29.18134653	23	32.67682672
11	29.83348129	24	32.73017408
12	30.38138944	25	32.77367042
13	30.83727305		



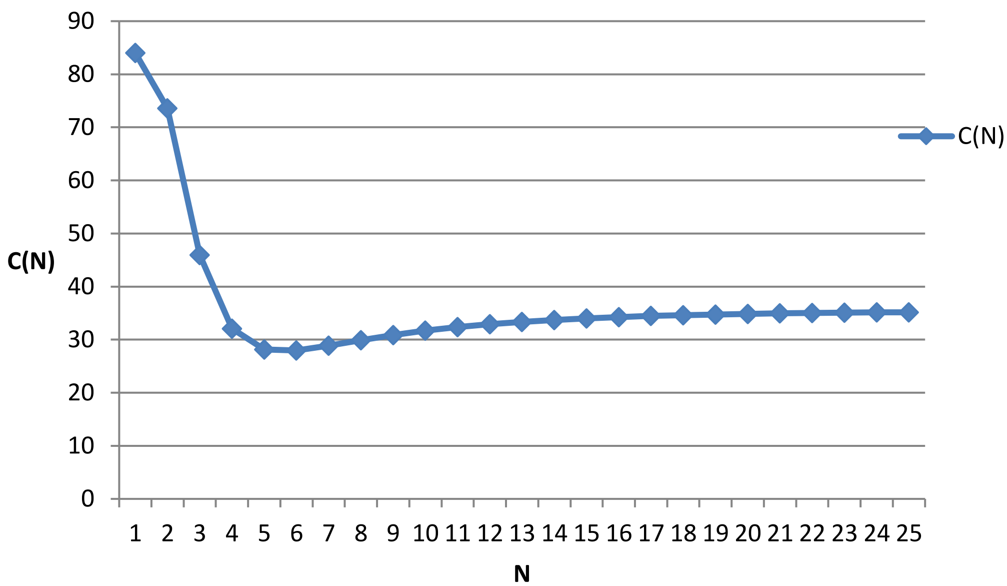
Graph 4.2: The long-run average cost per unit of time values against N)

Table: 4.3 **The long-run average cost per unit of time values against N**

For the hypothetical values C_r=40 , C=2500,C_w=100, p=0.75, α=0.95, β=-1.25,λ =20 ,µ=30 it could be computed the long-run average cost per unit of time is computed as follows:

N	C(N)	N	C(N)
1	84.00001243	14	33.71944128
2	73.58611416	15	34.01447954
3	45.92809734	16	34.25569396
4	32.0661705	17	34.45329227
5	28.13082822	18	34.61540516
6	**27.96156218**	19	34.74853682

7	28.85414766	20	34.85792052
8	29.91371556	21	34.94779277
9	30.87998092	22	35.02160322
10	31.69608256	23	35.0821752
11	32.36694457	24	35.13182933
12	32.9134193	25	35.17247931
13	33.35775454		



Graph 4.3: The long-run average cost per unit of time values against N

5. Conclusions

i. From the table 4.1 and graph 4.1, it is examined that the long-run average cost per unit time C(6) =26.01872761 is minimum for the given β= -0.95, α=0.95. Thus, we should replace the system at the time of 6th failure.

ii. From the table 4.2 and graph 4.2, it is observed that the long-run average cost per unit time C(6)=26.27475889 is minimum for the given β= -0.99, α=0.95. We should replace the system at the time of 6th failure. Thus, from above conclusions in (i) and (ii), it can be concluded that the long-run average cost per unit time increases with β.

iii. From the table 4.3 and graph 4.3, it is observed that the long-run average cost per unit time C(6)=27.96156218is minimum for the given β= -1.25, α=0.95. We should replace the system at the time of 6th failure.

iv. From the above conclusions (i) to (iii) , it is examined that the parameter 'β' is positively related with N, while negatively related with cost function. Similar conclusions may be drawn as 'α' decreases an increase in the number of failure, which coincides with the practical analogy and helps the decision maker for making an appropriate decision.

6. References

[1] A.D. Jerome Stanley, (1993), "On geometric processes and repair replacement problems", Microelectronics Reliability, Volume 33, Issue 4, , Pages 489-491.

[2] Amir Azaron,CahitPerkgoz Hideki Katagiri, Kosuke Kato and Masatoshi Sakawa, ,(2009), Multi-objective reliability optimization for dissimilar-unit cold-standby systems using a genetic algorithm", Computers & Operations Research, Volume 36, Issue 5, Pages 1562-1571.

[3] Barlow,R.E and Hunter,L.C., (1959), "Optimum Preventive Maintenance Policies",Operations Research, Volume 08, Pages 90-100.

[4] Balaguruswamy E, (1984) "Reliability Engineering", Tata Mc- Graw Hill, New Delhi, India.

[5] Barlow, R.E and Proschan.F., (1965), "Mathematical theory of reliability", Wiley, New York.

[6] Barlow, R.E and Proschan.F., (1975), "Statistical theory of reliability and life testing", Holt, Rinehart, Winston.

[7] Guan Jun Wang and Yuan Lin Zhang ,(2013),"Optimal repair–replacement policies for a system with two types of failures", European Journal of Operational Research, Volume 226, Issue 3, Pages 500-506.

[8] Guo Qing Cheng, Ling Li1,(2014),"An optimal replacement policy for a degenerative system with two-types of failure states", Journal of Computational and Applied Mathematics, Volume 261, Pages 139-145.