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Patterns and Secrets hidden in Pascal's Triangle

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Introduction

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1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

This may look like a neatly arranged stack of numbers, but it's actually a mathematical treasure trove. Indian mathematicians called it the Staircase of Mount Meru. In Iran, it's the Khayyam Triangle. And in China, it's Yang Hui's Triangle. To much of the Western world, it's known as Pascal's Triangle after French mathematician Blaise Pascal.

The pattern that generates Pascal's Triangle

Start with one and imagine invisible zeroes on either side of it. Add them together in pairs, and you'll generate the next row. Now do that again and again.

0 + 1 + 0

0 + 1 + 1 + 0

0 + 1 + 2 + 1 + 0

0 + 1 + 3 + 3 + 1 + 0

Binomial Expansion

Each row corresponds to the coefficients of a binomial expansion of the form $(x+y)^2$, where n is the number of the row.

Power	Binomial Expansion	Pascal's Triangle
0	$(x+y)^0=\mathbf{1}$	1
1	$(x+y)^1=\mathbf{1}x+\mathbf{1}y$	1,1
2	$(x+y)^2=\mathbf{1}x^2+\mathbf{2}xy+\mathbf{1}y^2$	1,2,1
3	$(x+y)^3=\mathbf{1}x^3+\mathbf{3}x^2y+\mathbf{3}xy^2+\mathbf{1}y^3$	1,3,3,1

Powers of 2

Add up the numbers in each row, and you'll get successive powers of 2.

Row Number	Addition of Numbers	In power of 2
0	1	2^0
1	1+1=2	2^1
2	1+2+1=4	2^2
3	1+3+3+1=8	2^3
4	1+4+6+4+1=16	2^4

Powers of 11

In each row, treat each number as part of a decimal expansion. It forms powers of 11 with respect to each row.

Row Number	Decimal Expansion	In power of 11
0	(1×1)=1	11^0
1	(1×10)+(1×1)=11	11^1
2	(1×100)+(2×20)+(1×1)=121	11^2
3	(1×1000)+(3×100)+(3×10)+(1×1)=1331	11^3
4	(1×10000)+(4×1000)+(6×100)+(4×10)+(1×1)=14641	11^4

Diagonals

- The first diagonal are all ones [Marked with Red color].
- The second diagonal contains natural numbers [Marked with Green color].
- The third diagonal contains the numbers called triangular numbers, because if you take that many dots, you can stack them into equilateral triangles. [Marked with Blue color].
- The fourth diagonal has the tetrahedral numbers, because similarly, you can stack that many spheres into tetrahedra [Marked with Purple color].

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Probability

Pascal's Triangle shows us the number of ways heads and tails can combine.

Number of Tosses	Possible Results [Grouped]	Pascal's Triangle
1	H T	1,1
2	HH HT,HT TT	1,2,1
3	HHH HHT,HTH,THH HTT,THT,TTH TTT	1,3,3,1

Conclusion

The patterns in Pascal's Triangle are a testament to the elegantly interwoven fabric of mathematics. And it's still revealing fresh secrets to this day.